RANDOM PULSATIONS IN A COARSELY DISPERSE FLUIDIZED BED

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A simple model of a uniform isotropic bed of large particles fluidized by a gas is offered which allows one to estimate the intensity of the chaotic translational and rotational motion of the particles. The influence of the pulsations on the observed macroscopic properties of the bed is discussed.

As is known, the main feature of fluidized systems, making them very attractive for practical use as the main working substance for heat exchangers, chemical-engineering apparatus, etc., is that the effective values of the parameters characterizing the intensity of transfer processes in them are usually several orders of magnitude higher than the analogous values for homogeneous fluids. This fact is connected in considerable measure with the occurrence of developed pulsation motions, both of the suspended particles and of the fluidizing medium, in a uniform fluidized bed or in the compact phase of a nonuniform bed. The theory of such "pseudoturbulent" motions was developed in [1] for beds of sufficiently fine particles, when the hydraulic forces of the interphase interaction are linear with respect to the relative velocity of the phases, while the interaction between particles is accomplished predominantly by random fields of the velocity and pressure of the continuous phase generated by all the particles, so that direct particle collisions can be neglected entirely in a first approximation.

Entirely different circumstances arise in fluidized beds of large particles; when the Reynolds number, which characterizes the displacement flow over one particle, is large compared with unity, the forces of interaction between the phases are nonlinear with respect to velocity, and the exchange of momentum and energy between particles is accomplished mainly through their collisions. In the limiting situation when the particles are very large and their pulsations are intense enough one can assume that the collisions lead to an approximately equilibrium distribution of the energy of chaotic motion over the translational and rotational degrees of freedom of the particles, as is assumed in [2], for example.

Let us consider a bed of spherical particles of radius a and density d_1 kept in the fluidized state by a homogeneous, ascending, gas stream of density d_0 and viscosity μ_0. For simplicity we assume the bed to be wide in the sense that the influence of the walls on its structure far from them can be neglected. We also neglect the regular circulation of the gas and the suspended material whose origin may be connected either with the retarding effect of the walls or with macroscopic instability of the homogeneous state under consideration. Then the mean values of the bed porosity $\langle \varepsilon \rangle$ and the gas flow $\langle \mathbf{Q} \rangle$ will depend only on the height $x > 0$ above the gas-distribution grid $x = 0$, but not on the transverse coordinates. In an investigation of local pulsations this dependence is also neglected. The mean gas velocity calculated for the free ("through") cross section of the bed is $\langle \mathbf{v} \rangle = \langle \varepsilon \rangle^{-1} \langle \mathbf{Q} \rangle$, while the mean particle velocity is $\langle \mathbf{w} \rangle = 0$.

In reality, the particles are involved in chaotic pulsating motion due to the interaction between the stream and porosity fluctuations [3], which also cause the appearance of fluctuations in the gas velocity and pressure. For the local instantaneous values of the gas and particle velocities and the bed porosity we write

$$\mathbf{v} = \langle \mathbf{v} \rangle + \mathbf{v}', \quad \mathbf{w} = \mathbf{w}', \quad \varepsilon = \langle \varepsilon \rangle + \varepsilon', \tag{1}$$

where the symbols with primes denote certain random functions of time and the coordinates which have zero means. The problem consists in finding the statistical characteristics of these random quantities in the form of functions of $\langle \varepsilon \rangle$ and $\langle \mathbf{Q} \rangle$ and the physical parameters of the particles and the gas.

For this purpose we investigate the equations of translational motion and rotation of one (test) particle, treated as a characteristic representative of the collection of suspended particles. The first of these equations has the form

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$$m d\mathbf{w}/dt = \mathbf{F} + \mathbf{F}_m + \mathbf{F}_c + m\mathbf{g}, \quad m = 4/3\pi a^3 d_1, \tag{2}$$

where F, F_m, and F_c are the force of the interaction with the carrier stream, directed along the stream, the transverse Magnus force, and the force acting on the test particle as a result of collisions with other particles. In the coordinate system being used the vector of gravitational acceleration is represented in components in the form g = (−g, 0, 0).

Because of the smallness of the density ratio d_0/d_1 we neglect the effect of the associated mass of the gas. Then to calculate F it is sufficient to consider the force F_s acting on the particle in a steady bed of motionless particles, which it is convenient to represent with the help of the two-term equation

$$\mathbf{F}_s = [\beta_1 K_1(\rho) + \beta_2 K_2(\rho) u]\,\mathbf{u}, \quad \mathbf{u} = \mathbf{v} - \mathbf{w}, \tag{3}$$

where β_1 and β_2 are certain coefficients and $K_1(\rho)$ and $K_2(\rho)$ are functions of the local bed concentration $\rho = 1-\varepsilon$. A large number of empirical equations have been suggested for these quantities; here we use Ergun's equation [4], valid for $\varepsilon \leqslant 0.7$, in accordance with which

$$\beta_1 = 50\pi a\mu_0, \quad \beta_2 = \frac{3{,}5}{3}\pi a^2 d_0, \quad K_1(\rho) = \frac{\rho}{\varepsilon^2}, \quad K_2(\rho) = \frac{1}{\varepsilon}. \tag{4}$$

To obtain a representation for the force acting in a bed of freely pulsating particles we expand F_s in a Taylor series with respect to the fluctuations introduced in (1). Being confined to terms of first order with respect to the fluctuations, for the average force $\langle F \rangle$ and its pulsation F' we obtain

$$\begin{gathered} \langle \mathbf{F} \rangle = \langle \mathbf{F}_s \rangle + \beta_1 (K_1^* \langle \rho'\mathbf{u}' \rangle + {}^1\!/_2 K_1^{**} \langle \rho'^2 \rangle \langle \mathbf{u} \rangle) + \\ + \beta_2 \{K_2 [\langle (\mathbf{u}_0\mathbf{u}')\,\mathbf{u}' \rangle + {}^1\!/_2 \langle u'^2 \rangle \mathbf{u}_0 - {}^1\!/_2 \langle (\mathbf{u}_0\mathbf{u}')^2 \rangle \mathbf{u}_0] + \\ + K_2^* [\langle u \rangle \langle \rho'\mathbf{u}' \rangle + \langle \rho' (\mathbf{u}_0\mathbf{u}') \rangle \langle \mathbf{u} \rangle] + {}^1\!/_2 K_2^{**} \langle \rho'^2 \rangle \langle u \rangle \langle \mathbf{u} \rangle\}, \\ \mathbf{F}' = (\beta_1 K_1 + \beta_2 K_2 \langle u \rangle)\,\mathbf{u}' + \beta_2 K_2 (\mathbf{u}_0\mathbf{u}') \langle \mathbf{u} \rangle + \\ + (\beta_1 K_1^* + \beta_2 K_2^* \langle u \rangle) \langle \mathbf{u} \rangle \rho', \quad \mathbf{u}_0 = \langle \mathbf{u} \rangle / \langle u \rangle. \end{gathered} \tag{5}$$

Here we introduce the fluctuations $\rho' = -\varepsilon'$ in the volume concentration of the disperse phase in the bed, while

$$\langle \mathbf{F}_s \rangle = (\beta_1 K_1 + \beta_2 K_2 \langle u \rangle) \langle \mathbf{u} \rangle, \quad K_j = K_j(\langle \rho \rangle), \tag{6}$$

where $\langle F_s \rangle$ represents the force acting in a uniform stationary bed whose porosity coincides with the mean porosity $\langle \varepsilon \rangle$ of the fluidized bed, while the asterisk denotes differentiation with respect to $\langle \rho \rangle$. We represent the Magnus force F_m in the form [5]

$$\mathbf{F}_m = \beta_3 \varepsilon^{-1} (\boldsymbol{\lambda} \times \mathbf{u}), \quad \beta_3 = \frac{8}{3}\pi a^3 d_0. \tag{7}$$

The factor ε^{-1} in the expression for the coefficient is introduced phenomenologically here by analogy with the representation of the second term in (3), as if the displacement streamline flow led to an increase of ε^{-1} times in the stresses at the surface of the particle responsible for the appearance of the Magnus force.

The quantity λ in (7) represents the angular velocity of rotation of the particle; its mean value is equal to zero owing to the symmetry of the problem. It is clear that the Magnus force is important only at a high relative gas velocity u; it was evidently first used in [2, 6] in an analysis of fluidized and generally concentrated disperse systems. From (7), by analogy with the derivation of (5) from (3), we have

$$\langle \mathbf{F}_m \rangle = 0, \quad \mathbf{F}_m' = \beta_3 \langle \varepsilon \rangle^{-1} (\boldsymbol{\lambda}' \times \langle \mathbf{u} \rangle). \tag{8}$$

Regarding the force F_c, connected with direct particle collisions, it is only known that it should be represented in the form of a sum of the forces of collisional interactions with individual particles, which differ from zero only during the small time intervals of such interactions.

By analogy with (2), we write the equation for the rotation of the particle in the form

$$Id\lambda/dt = \mathbf{M} + \mathbf{M}_c, \ I = {}^8/_{15}\pi a^5 d_1, \tag{9}$$

where M and M_c are the moments acting on the test particle on the part of the carrier stream and the other particles with which it collides. Unfortunately, almost nothing is known either about M_c or (for large particles) about the moment M.

In such a situation no consistent analysis of the stochastic equations (2) and (9) proves to be possible for large particles. In this respect the coarsely disperse system under consideration differs importantly from suspensions of fine particles when, first, one can either entirely ignore the Magnus force [and the necessity of studying Eq. (9)] or express M with the help of a linear equation through the curl of the velocity of the suspension and the angular velocity of the particle, and second, one can neglect direct particle collisions by assuming that they interact by means of the continuous phase. In the case under consideration, on the contrary, collisions are very important and such a "collisionless" approximation is invalid. To overcome the serious difficulties which arise, therefore, one must employ some additional considerations which will make it possible not to treat Eqs. (2) and (9) in their original form.

For large enough particles the exchange of momentum and energy in collisions is comparable in magnitude with the momentum and energy of the individual colliding particles. Therefore, the relaxation time of the suspended particles is on the same order as the time required for a small number of successive collisions. Considering that the latter time is quite short in a concentrated fluidized bed (up to $\rho \approx 0.1$), henceforth we will be confined to an analysis of only that limiting equilibrium state in which the principle of the equilibrium distribution of the pulsation energy of the particles over the degrees of freedom must clearly be satisfied. We then have the relations

$$\begin{gathered} m\langle w^2\rangle = I\langle\lambda^2\rangle, \\ \langle w_x^2\rangle = \langle w_y^2\rangle = \langle w_z^2\rangle = \frac{1}{3}\langle w^2\rangle, \ \langle\lambda_x^2\rangle \\ = \langle\lambda_y^2\rangle = \langle\lambda_z^2\rangle = \frac{1}{3}\langle\lambda^2\rangle, \end{gathered} \tag{10}$$

in accordance with which it is sufficient to find the average energy of the translational motion of the particles to determine their root-mean-square velocities. Since $\mathbf{w}' = \mathbf{w}$ and $\lambda' = \lambda$, we drop the primes in the designations of the pulsations $\mathbf{w}'$ and λ' in (10) and below.

Because the problem is steady-state, the mean pulsation energy does not depend on time, and from (2) and (9), after scalar multiplication by $\mathbf{w}$ and λ, respectively, and averaging, we obtain

$$\langle \mathbf{F}'\mathbf{w}\rangle + \langle \mathbf{F}'_m\mathbf{w}\rangle + \langle \mathbf{F}'_c\mathbf{w}\rangle = 0, \ \langle \mathbf{M}\lambda\rangle + \langle \mathbf{M}_c\lambda\rangle = 0, \tag{11}$$

with the first terms in (11) describing the mean work of the forces F' and F_m' and the moment M on the random movements of the particle per unit time while the last terms characterize the energy dissipation during particle collisions. This dissipation is due to the inelasticity of the collisions, as a result of which part of the energy goes into heat within the particles, to the abrupt change in particle velocity during the collisions, and to the additional viscous dissipation of energy in the gas connected with it. With good accuracy one can neglect these effects, as well as the last terms in (11).

The latter means that the energy of particle pulsations in the fluidized bed under consideration will be the same as that in some fictitious bed with the same average parameters but in which particle collisions are absent. It is clear that the pulsations in the particle velocities in such a fictitious bed (denoted by capitolized symbols to avoid confusion) satisfy the following equations:

$$m d\mathbf{W}/dt = \mathbf{F}' + \mathbf{F}'_m, \ Id\Lambda/dt = \mathbf{M}. \tag{12}$$

As before, the moment M is unknown. For simplicity below, however, we use an "inertialess" approximation, according to which we neglect the particle's inertia when studying both its rotation and its translational motion. Then the terms on the left sides of (12) vanish and the second equation gives simply M = 0, which is possible only when the vector rot V/2, characterizing the local vorticity of the gas near the particle, coincides with the angular velocity Λ. Within the framework of the inertialess approximation the time dependence of the fluctuations is unimportant, and they can be formally treated as steady random fields. We obtain the equations corresponding to this approximation, which the pulsation frequency ω must satisfy, from a comparison of the terms on the left and right sides of the first equation of (12) with allowance for the expressions for the coefficients β_1 and β_2 in (4):

$$\omega \ll 100 \frac{\nu_0}{a^2} \frac{d_0}{d_1}, \quad \omega \ll \frac{\langle u \rangle}{a} \frac{d_0}{d_1}, \quad \nu_0 = \frac{\mu_0}{d_0}. \tag{13}$$

If inequalities (13) are violated then the results obtained with the particle inertia neglected have only an order-of-magnitude character.

Using what was said above and the expressions for F' and F_m' in (5) and (8), from (12) we obtain the following equations for the fluctuations in the parameters of the fictitious collisionless bed:

$$(\beta_1 K_1 + \beta_2 K_2 \langle u \rangle) \mathbf{U}' + \beta_2 K_2 (\mathbf{u}_0 \mathbf{U}') \langle \mathbf{u} \rangle = -\beta_1 K_1^* \langle \mathbf{u} \rangle \rho' \tag{14}$$

$$-\beta K_2^* \langle u \rangle \langle \mathbf{u} \rangle \rho' - \beta_3 \langle \varepsilon \rangle^{-1} (\mathbf{\Lambda} \times \langle \mathbf{u} \rangle), \quad \mathbf{\Lambda} = \operatorname{rot} \mathbf{V}'/2.$$

To express W and Λ only through the fluctuation in the concentration one must find a representation for the pulsation velocity V' of the gas, for which, in accordance with the general method in [1], one would have to consider the equations of motion of the gas in the gaps between particles. Such an analysis is very cumbersome; but with the accuracy adopted here it can be simplified considerably by estimating V' from the condition of constancy of the local gas flow, i.e., from the equality $Q' = \varepsilon' \langle v \rangle + \langle \varepsilon \rangle V' = 0$. This gives

$$\mathbf{V}' = -\langle \varepsilon \rangle^{-1} \varepsilon' \langle \mathbf{v} \rangle = \langle \varepsilon \rangle^{-1} \rho' \langle \mathbf{u} \rangle, \quad \mathbf{V}' = \mathbf{U}' + \mathbf{W}. \tag{15}$$

The approximation leading to (15) is equivalent to the assumption that the influence of stresses in the gas on the particle pulsation is weak, which was already partly used in the derivation of (5), a consequence of which is the total neglect of fluctuations in the pressure and in the horizontal components of the gas velocity. This is indirectly confirmed by the analysis in [1], according to which these fluctuations do not much affect the pulsations of fine enough particles even when they are fluidized by drops of liquids.

Now let us solve Eqs. (14) and (15). Since $\langle \mathbf{u} \rangle = (\langle u \rangle, 0, 0)$ in the components, from (15) and the second equation of (14) we have the following representation for Λ:

$$\mathbf{\Lambda} = \tfrac{1}{2} \langle \varepsilon \rangle^{-1} \langle u \rangle (0, \partial\rho'/\partial z, -\partial\rho'/\partial y). \tag{16}$$

Then for the vector $\mathbf{\Lambda} \times \langle \mathbf{u} \rangle$ we obtain

$$\mathbf{\Lambda} \times \langle \mathbf{u} \rangle = -\tfrac{1}{2} \langle \varepsilon \rangle^{-1} \langle u \rangle^2 (0, \partial\rho'/\partial y, \partial\rho'/\partial z), \tag{17}$$

so that the first equation in (14) leads to the equalities

$$\begin{gathered}(\beta_1 K_1 + 2\beta_2 K_2 \langle u \rangle) U_x' = -\beta_1 K_1^* \langle u \rangle \rho' - \beta_2 K_2^* \langle u \rangle^2 \rho', \\ (\beta_1 K_1 + \beta_2 K_2 \langle u \rangle) \mathbf{U}_\perp' = \tfrac{1}{2} \beta_3 \langle \varepsilon \rangle^{-2} \langle u \rangle^2 \nabla_\perp \rho',\end{gathered} \tag{18}$$

where $U_\perp'$ is the projection of U' onto the horizontal plane in which the gradient operator $\nabla_\perp \rho' = (0, \partial\rho'/\partial y, \partial\rho'/\partial z)$ operates.

From (15) and (18) it is easy to obtain representations for the components of the vector W. In accordance with the data indicated above, the total energy $E = \frac{1}{2}(m\langle W^2 \rangle + \langle \Lambda^2 \rangle)$ of the pulsating motion of a particle is interesting. We have

$$\frac{\langle \Lambda^2 \rangle}{\langle u \rangle^2} = \frac{1}{4 \langle \varepsilon \rangle^2} \langle |\nabla_\perp \rho'|^2 \rangle, \tag{19}$$

$$\frac{\langle W^2 \rangle}{\langle u \rangle^2} = \left(\frac{1}{\langle \varepsilon \rangle} + \frac{\beta_1 K_1^* + \beta_2 K_2^* \langle u \rangle}{\beta_1 K_1 + 2\beta_2 K_2 \langle u \rangle} \right)^2 \langle \rho'^2 \rangle + \frac{1}{4 \langle \varepsilon \rangle^2} \left(\frac{\beta_3 \langle \varepsilon \rangle^{-1} \langle u \rangle}{\beta_1 K_1 + \beta_2 K_2 \langle u \rangle} \right)^2 \langle |\nabla_\perp \rho'|^2 \rangle. \tag{20}$$

For the final determination of E we must find explicit expressions for $\langle \rho'^2 \rangle$ and $\langle |\nabla_\perp \rho'|^2 \rangle$. The latter is easy to do by using the spectral theory of the random concentration of a disperse system developed in [7], from which

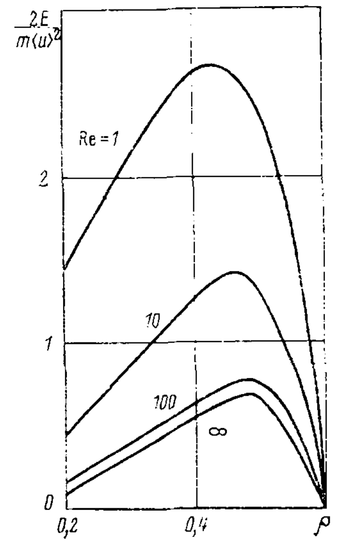

Fig. 1. Dependence of particle pulsation energy on ρ at different Reynolds numbers.

$$\langle \rho'^2 \rangle = \langle \rho \rangle^2 \left(1 - \frac{\langle \rho \rangle}{\rho_*}\right), \quad \langle |\nabla_\perp \rho'|^2 \rangle = \frac{2}{5}\left(\frac{9\pi}{2}\right)^{2/3} \frac{\langle \rho \rangle^{8/3}}{a^2}\left(1 - \frac{\langle \rho \rangle}{\rho_*}\right). \tag{21}$$

From (19)-(21) we have

$$\frac{2E}{m\langle u \rangle^2} = \langle \rho \rangle^2 \left(1 - \frac{\langle \rho \rangle}{\rho_*}\right)\left\{\left(\frac{1}{\langle \varepsilon \rangle} + \frac{\beta_1 K_1^* + \beta_2 K_2^* \langle u \rangle}{\beta_1 K_1 + 2\beta_2 K_2 \langle u \rangle}\right)^2 + \frac{1}{10}\left(\frac{9\pi}{2}\right)^{2/3} \frac{\langle \rho \rangle^{2/3}}{\langle \varepsilon \rangle^2}\left[\frac{2}{5} + \left(\frac{a^{-1}\beta_3 \langle \varepsilon \rangle^{-1} \langle u \rangle}{\beta_1 K_1 + \beta_2 K_2 \langle u \rangle}\right)^2\right]\right\}, \tag{22}$$

which also determines the rms components of the translational and angular velocities of the particles in accordance with (10).

If we use Eqs. (4) and (7) we obtain from (22), after a simple calculation,

$$\frac{2E}{m\langle u \rangle^2} = \left(\frac{\langle \rho \rangle}{\langle \varepsilon \rangle}\right)^2 \left(1 - \frac{\langle \rho \rangle}{\rho_*}\right)\left\{\left(1 + \frac{1 + \langle \rho \rangle + 0.0233\,\mathrm{Re}}{\langle \rho \rangle + 0.0466\,\mathrm{Re}}\right)^2 + 0.58\langle \rho \rangle^{2/3}\left[0.4 + \left(\frac{0.0534\mathrm{Re}}{\langle \rho \rangle + 0.0233\mathrm{Re}}\right)^2\right]\right\}, \tag{23}$$

where we introduced the modified Reynolds number

$$\mathrm{Re} = a\nu_0^{-1} \langle \varepsilon \rangle \langle u \rangle = a\nu_0^{-1} \langle Q \rangle. \tag{24}$$

As Re → 0 and Re → ∞ we obtain the asymptotic equations

$$\frac{2E}{m\langle u \rangle^2} = \left(\frac{\langle \rho \rangle}{\langle \varepsilon \rangle}\right)^2 \left[\left(2 + \frac{1}{\langle \rho \rangle}\right)^2 + 0.232\langle \rho \rangle^{2/3}\right]\left(1 - \frac{\langle \rho \rangle}{\rho_*}\right),$$
$$\frac{2E}{m\langle u \rangle^2} = \left(\frac{\langle \rho \rangle}{\langle \varepsilon \rangle}\right)^2 (2.25 + 3.28\langle \rho \rangle^{2/3})\left(1 - \frac{\langle \rho \rangle}{\rho_*}\right). \tag{25}$$

The dependence of the particle pulsation energy on $\langle\rho\rangle$ at different Re and $\rho_* = 0.6$ is shown in Fig. 1, from which it is seen that this quantity has a single maximum at $\langle\rho\rangle = \rho_m$ (Re). Remember that Ergun's equation [4], used in the calculations, gives satisfactory results only in the region of $\langle\rho\rangle \gtrsim 0.3$.

In connection with the indicated theoretical difficulties and, in particular, the necessity of an explicit allowance for the collisional force F_c and the moment M_c, it does not seem possible to determine the statistical characteristics of the fluctuations in gas velocity within the framework of the simplified model under consideration.

Above we used the "mean" characteristics $<\varepsilon>$ and $<Q>$ of a fluidized bed, but didn't discuss their connection with the real observed values of the porosity ε_0 and the gas flux (fluidization velocity) Q_0. Obviously, $<\varepsilon> \equiv \varepsilon_0$. But the flux $< Q >$, which must be used in the analysis being conducted, differs from Q_0. In fact, from the definition of the local flux Q we have

$$\mathbf{Q}_0 = \langle \varepsilon \rangle \langle \mathbf{v} \rangle + \langle \varepsilon' \mathbf{v}' \rangle = \langle \mathbf{Q} \rangle - \langle \rho' \mathbf{v}' \rangle, \tag{26}$$

where $-<\rho' v'>$ is the additional gas flux due to the fluctuations in the parameters of the bed. This flux coincides in order of magnitude with

$$-\langle \rho' \mathbf{V}' \rangle = (\rho_0/\varepsilon_0)^2 (1 - \rho_0/\rho_*) \langle \mathbf{Q} \rangle \tag{27}$$

[here we allow for Eq. (15) for V']. It is not hard to show by direct calculations that this quantity is an order of magnitude lower than $< Q >$ (its maximum value is about 9% of $<Q>$ and is reached at $\rho_0 = \frac{2}{3}\rho_* = 0.4$ if one uses $\rho_* = 0.6$). Hence we can neglect the difference between $<Q>$ and Q_0 within the framework of the approximate theory under consideration.

Then from the first equation of (5) it follows that the average force $< F >$ of hydraulic resistance of the fluidized bed differs from the force F_S of resistance of a stationary bed of the same porosity, "ideal" in the sense that porosity fluctuations are absent from it. One can show that the effect of a decrease in the hydraulic resistance of the fluidized bed in comparison with a stationary bed occurs. This effect was analyzed rigorously in the limiting case of $Re \to 0$ in [1], where satisfactory agreement with the experimental data was achieved. A brief discussion of this effect in connection with the existence of porosity fluctuations in real fluidized beds is given in [8]. We note that the difference in the hydraulic resistances of stationary and fluidized beds may also be connected with the fact that in the first case the force acting on an individual particle fluctuates while in the second case the velocity of the gas near it and the velocity of the particle itself fluctuate [9].

Equation (6) also describes the resistance of a stationary bed having a nonuniform porosity, which also differs from the resistance of an ideal bed without fluctuations. In this connection we note that the force F_S introduced in (3) must pertain just to an ideal bed, whereas Ergun's equation gives the connection between the bed and the flow Q_0 in a bed with random particle packing in which, of course, there are porosity fluctuations. This difference can be neglected, however, in view of the empirical character of this equation and the approximate nature of the theory itself.

It is easy to analyze the influence of particle pulsations on the macroscopic structure of the bed or its compact phase. The tensor of the momentum flux transported by the particles in their pulsating motion is isotropic under the condition of equal distribution of the pulsation energy over the degrees of freedom. Its only independent component can be identified with the "pressure" of the disperse phase,

$$P = \frac{1}{3} nm \langle w^2 \rangle = \frac{1}{3} \rho d_1 \langle w^2 \rangle . \tag{28}$$

The pressure P as a function of w has a single maximum. The connection in (28) between the pressure of the disperse phase and the rms velocity of the pulsations was verified experimentally in [10], where they also confirmed the isotropic nature of the pressure, and hence Eq. (10).

Neglecting the Archimedes force, as in (2), the equation of conservation of particle momentum under steady conditions can be written in the form

$$\nabla P = n(\langle \mathbf{F} \rangle - mg). \tag{29}$$

This equation can be obtained rigorously either from the kinetic equation for the particle velocity distribution function, as in [1], or from (2) if one multiplies (2) by n, writes the time derivative in (2) in the form of a convective derivative, uses the condition of conservation of particle mass, and conducts the averaging in the same way as in the derivation of the Reynolds equations for a turbulized fluid.

With allowance for the expressions for $<F>$ and $<w^2>$, Eqs. (28) and (29) allow one to analyze various problems on the height distribution of the disperse phase of a fluidized bed, the distribution near rising bubbles, etc. Moreover, the results obtained are needed in a calculation of the effective transfer coefficients in a bed (e.g., see the investigation of the electrical conductivity of a fluidized bed in [11], where these very results were used).

For determinacy the subject above was a uniform fluidized bed, but it is easy to see that all the equations obtained are also valid for a nonuniform bed if they refer only to its compact phase. According to the well-known two-phase theory of fluidization, a nonuniform bed can be described as a combination of a compact phase and a phase of bubbles propagating in it, with the state of the compact phase being assumed to be close to the state of initial fluidization, so that it would seem that the quantity ρ must always be close to ρ_*. In reality, as the tests in [12] show, coarsely disperse fluidized beds deviate considerably from the requirements of the two-phase theory, and their compact phase can be characterized by values of ρ lying in a very broad range.

In conclusion, it should be emphasized once again that the proposed theory pertains to the state of developed fluidization with sufficiently intense pulsation motions of the particles. In the case of incompletely fluidized or steady beds, the friction between particles rolling over one another, the thrust forces arising in such rolling, analogous in some measure to the normal stresses in flows of non-Newtonian fluids, and so forth play a major role, which was entirely ignored above. Some of the ideas discussed in [6] will evidently prove useful when introduced into an analysis of these effects.

NOTATION

a and m, particle radius and mass; d_0 and d_1, gas and particle densities; μ and ν, dynamic and kinematic viscosities of the gas; ε and $\rho = 1 - \varepsilon$, porosity of bed and concentration of solid particles in bed; x, y, z, t, coordinates and time; v, w, **u** = v − w, velocities of gas and particles and their relative velocity, respectively; Q, volumetric gas velocity; F, F_m, F_c, F_s, force of interaction of particles with carrier stream, transverse Magnus force, force acting on a particle as a result of collisions with other particles, and force of interaction of particles with gas stream in a stationary bed; g, acceleration of gravity; β_1 and β_2, coefficients; $K_1(\rho)$ and $K_2(\rho)$, functions of local bed concentration; λ, angular velocity of particle rotation; I, moment in inertia of a particle about an axis passing through its center; M and M_c, moments acting on a particle on the part of the carrier stream and other particles with which it collides; E, total energy of pulsating motion of a particle; n, number concentration of particles; angle brackets are used in designations of average quantities; symbols with primes are some random functions of time and the coordinates having zero means.

LITERATURE CITED

1. Yu. A. Buevich (Buyevich), J. Fluid Mech., 49, 489 (1971); 52, 345 (1972); 56, 313 (1972).
2. M. A. Gol'dshtik, Zh. Prikl. Mekh. Tekh. Fiz., No. 6, 106 (1972).
3. Yu. A. Buevich, Zh. Prikl. Mekh. Tekh. Fiz., No. 6, 35 (1966).
4. S. Ergun, Chem. Eng. Prog., 48, 89 (1952).
5. M. A. Gol'dshtik and B. N. Kozlov, Zh. Prikl. Mekh. Tekh. Fiz., No. 6 , 149 (1968).
6. M. A. Gol'dshtik and B. N. Kozlov, Zh. Prikl. Mekh. Tekh. Fiz., No. 4, 67 (1973).
7. Yu. A. Buevich, Zh. Prikl. Mekh. Tekh. Fiz., No. 6, 60 (1970).
8. O. M. Todes, Teor. Osn. Khim. Tekhnol., 1, 515 (1967).
9. T. S. Lundgren, J. Fluid Mech., 51, 273 (1972).
10. H. P. Meissner and C. L. Kusik, Can. J. Chem. Eng., 48, 349 (1970).
11. V. A. Borodulya and Yu. A. Buevich, Inzh.-Fiz. Zh., 35, No. 5 (1978).
12. D. Geldart and R. R. Cranfield, Chem. Eng. J., 3, 211 (1972).